Inverse meson mass ordering in color-flavor-locking phase of high density QCD: erratum

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Abstract

We correct a mistake in the calculation of meson masses at large baryon chemical potential μ made in Ref. [1].

The values of the meson masses reported in Ref. [1] are not correct. The correct answer for the meson masses and mixings are given by Eqs. (45) and (46) in Ref. [1] with $C = 2c/f_\pi^2$, and

$$c = \frac{3\Delta^2}{2\pi^2}, \tag{1}$$

where Δ is the value of the superconducting gap at the Fermi surface. To leading order in perturbation theory, the constant denoted in Ref. [1] as c' vanishes. Parametrically, the meson masses are of order $m_q\Delta/\mu$ instead of m_q as reported in Ref. [1]. Moreover, since c' vanishes to the leading order, a truly inverse mass hierarchy emerges instead of a partial one previously reported. This means the inverse hierarchy anticipated by the heuristic argument following Eq. (46) in Ref. [1] is indeed realized.

Technically, the mistake of our previous calculation of the masses came from the failure to take into account the fact that the quark mass insertion in a quark line changes the quark mass shell from $E = |\boldsymbol{p}| - \mu$ to $E = -|\boldsymbol{p}| - \mu$ [2]. As a result, an almost on-shell quark near the Fermi surface becomes an off-shell particle with virtuality 2μ.

The correct computation of the constants c and c' follows the same methodology of Ref. [1] of matching vacuum energies and involves the evaluation of two Feynman diagrams in Fig. 1, which both make leading-order contributions to the shift of the vacuum energy. Note that the gap insertions, which reverse the direction of the quark lines in Fig. 1, occur only on the propagators which are near the particle mass shell. The lower part of the first diagram between M insertions corresponds to the one-loop quark self-energy and is equal, by the gap equation, to a quantity commonly referred to as the "anti-particle gap". The latter is, however, not gauge invariant even to leading order in perturbation theory. Consequently, the first diagram is not gauge-fixing independent on its own. However, the sum of the two

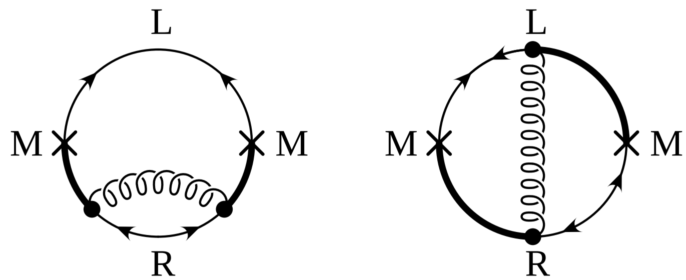

Figure 1: These two diagrams (together with their parity and charge conjugation mirrors) contribute to the shift of vacuum energy to the leading order in g. Thin lines denote quarks which are almost on-shell near the Fermi surface (virtualities of order Δ). Thick lines correspond to the off-shell quarks (virtualities close to 2μ).

diagrams is indeed gauge-fixing independent, which is a consequence of Ward-Takahashi identity and can be verified by a direct calculation.

The following comments concerning the result (1) are in order. The contribution of the *magnetic* gluon exchange cancels between the two diagrams. This happens because emission of a magnetic gluon by an on-shell quark is suppressed by parity conservation. As a result, c is proportional to Δ^2 instead of $\Delta^2 \log(\mu/\Delta)$. The remaining electric gluon exchange is suppressed at small gluon momenta because of the angular momentum conservation and the requirement that the helicity flip must occur in the diagram. This removes a collinear divergence which would have given rise to an extra factor of $1/\log g$ [3].

References

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